A Liouville Theorem on the PDE $\det(f_{i\bar{j}}) = 1$

Abstract

Let f be a smooth plurisubharmonic function which solves

$$\det(f_{i\bar{j}}) = 1 \qquad \text{in } \Omega \subset \mathbb{C}^n.$$

Suppose that the metric $\omega_f = \sqrt{-1} f_{i\bar{j}} dz_i \wedge d\bar{z}_j$ is complete and f satisfies the growth condition

$$C^{-1}(1 + |z|^2) \le f \le C(1 + |z|^2), \quad as \quad |z| \to \infty.$$

for some $C > 0$, then f is quadratic.